Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
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3. Answer all questions.

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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

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See next page

Question 1   **(5 marks)**

Rewrite the following as single a logarithm:

(a) $1 + 2\log(x) - \log(1 + y)$ (2 marks)

(b) $2\ln(3) + \ln(16) - 2\ln\left(\frac{6}{5}\right)$ (3 marks)

Question 2   **(4 marks)**

Let $y = x \log_e(3x)$

(a)   Find $\frac{dy}{dx}$.   (2 marks)

(b)   Hence, calculate $\int_1^2 (\log_e(3x) + 1)dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer.   (2 marks)

Question 3   **(3 marks)**

Find the maximum value of $y = x^2 e^{-3x}, x > 0$. No test for the nature of the stationary point is required.

Question 4   **(3 marks)**

The area enclosed between the curve $y = \dfrac{2}{3x-1}$, the lines $x = 1$ and $x = 3$ and the $x - axis$ can be written as $\log_e B$ square units. Calculate the value of B.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name_____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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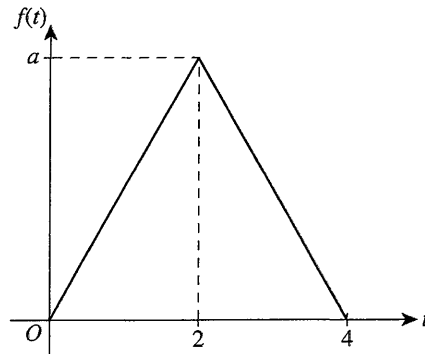
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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 5 **(5 marks)**

The length of time, T minutes, that a Mathematics Methods student takes to answer an examination question is a random variable defined by the probability function whose graph is shown below.

(a) State the value of a so that f is a probability density function. (1 mark)

(b) Hence, state the rule for the probability function, f. (2 marks)

(c) Find $P(T \le 1)$. (2 marks)

See next page

Question 6 **(2 marks)**

In a large population of fish, the proportion of angel fish is 0.25. Let $\hat{P}$ be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of $\hat{P}$ is less than or equal to 0.01.

Question 7 **(10 marks)**

The travelling time for shuttle buses from train station A to train station B (Route AB) is found to be normally distributed with a mean arrival time of 3 hours and 45 minutes and standard deviation of 8 minutes.

(a) Find the probability that a bus on Route AB will complete its journey in under 3 hours and 50 minutes. Give your answer correct to three decimal places.

(2 marks)

(b) Hence, find the probability that the same bus will be no earlier than 5 minutes from its mean arrival time. Give your answer correct to three decimal places.

(2 marks)

(c) 90% of the shuttles will complete the Route AB within t minutes. Find t correct to the nearest minute. (2 marks)

(d) Within what range of times could we be 95% confident of completing Route AB?

(2 marks)

A random sample of the travel time to complete Route AB is taken from the time sheets of 10 shuttles.

(e) What is the probability that of the 10 times selected, 6 of them will have completed the journey in under 3 hours and 50 minutes? Give your answer correct to four decimal places. (2 marks)

See next page

Question 8 (3 marks)

The probability density function for a continuous random variable X has the rule

$$f(x) = \begin{cases} a\sin(\pi x) & for \quad 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(a) Find the value of a. (1 mark)

(b) Find $P\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right)$. (2 marks)

Question 9 (3 marks)

Jo, a physiotherapist, is undertaking a study of her patients' exercising habits. She decides to ask every eighth patient about their exercise habits.
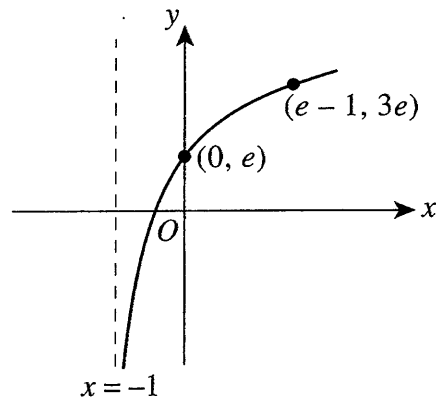
(a) What type of sample is this? (1 mark)

(b) Is this method of sampling valid? Justify your answer. (2 marks)

Question 10 **(3 marks)**

The graph of $y = a \log_e(x - b) + c$ is shown below.

Calculate the exact values of b, c and a.

Question 11 (4 marks)

During the COVID-19 pandemic Jeff's Gin Distillery that once produced ethanol for gin, now uses it to produce sanitizer. It fills bottles of sanitizer uniformly to between 245 ml and 253 ml.
The label on the bottle states that it holds 250ml.

(a) Determine the probability that a bottle selected randomly from the production line contains less than the labelled amount. (1 marks)

(b) Calculate the mean and standard deviation of the amount of sanitizer in the bottles. (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2020
TEST 4

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____ • Solutions • _____

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1 **(5 marks)**

Rewrite the following as single a logarithm:

(a) $1 + 2\log(x) - \log(1 + y)$ (2 marks)

$$= \log 10 + \log x^2 - \log(1+y)$$

$\checkmark$ $\left(\begin{array}{l}\text{convert } 1 \text{ to } \log 10 \\ \text{\& uses } 1 \log \text{ law}\end{array}\right)$

$$= \log 10x^2 - \log(1+y)$$

$$= \log\left(\frac{10x^2}{1+y}\right)$$

$\checkmark$ (Answ)

(b) $2\ln(3) + \ln(16) - 2\ln(\frac{6}{5})$ (3 marks)

$$= \ln 3^2 + \ln 16 - \ln\left(\frac{6}{5}\right)^2$$

$\checkmark$ $(\text{brings } 2 \text{ to index})$

$$= \ln 9 + \ln 16 - \ln\frac{36}{25}$$

$$= \ln\left(\frac{9 \times 16}{\frac{36}{25}}\right)$$

$\checkmark$ $\left(2 \times \log \text{ laws}\right)$

$$= \ln \frac{9 \times 16^4 \times 25}{36_1}$$

$$= \boxed{\ln 100}$$

$\checkmark$ $\left(\begin{array}{l}\text{Answ} \\ \text{as single logarithm}\end{array}\right)$

5

Question 2   **(4 marks)**

Let $y = x \log_e(3x)$ $u'v + uv'$

(a)   Find $\frac{dy}{dx}$.   **(2 marks)**

$$\frac{dy}{dx} = \ln(3x) + x \cdot \frac{1}{x} \quad \checkmark \quad (\text{uses product Rule})$$

$$= \boxed{\ln(3x) + 1} \quad \checkmark \quad (\text{simplifies})$$

(b)   Hence, calculate $\int_1^2 (\log_e(3x) + 1)dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer.   **(2 marks)**

$$\int_1^2 \ln(3x) + 1 \, dx = \left[x \ln(3x) \right]_1^2 \quad \checkmark \quad (\text{integrates})$$

$$= 2\ln 6 - \ln 3$$

$$= \ln 36 - \ln 3$$

$$= \boxed{\ln 12} \quad \checkmark \quad (\text{ANSW in } \ln(a) \text{ form})$$

4

Question 3 u v **(3 marks)**

Find the maximum value of $y = x^2 e^{-3x}$, $x > 0$. No test for the nature of the stationary point is required.

$$u'v + uv'$$

$$\text{Max when } \frac{dy}{dx} = 0$$

$$2x e^{-3x} + x^2(-3)e^{-3x} = 0 \quad \checkmark \left(\frac{dy}{dx} = 0\right)$$

$$x e^{-3x}(2 - 3x) = 0$$

$$x = 0 \text{ or } \boxed{x = \tfrac{2}{3}} \text{ but as } x > 0$$
$$\checkmark \text{ (solves for } x\text{)}$$

$$\therefore y = \left(\tfrac{2}{3}\right)^2 e^{-3\left(\tfrac{2}{3}\right)}$$

$$\therefore \text{ Max value } \boxed{\frac{4}{9e^2}} \quad \checkmark \text{ (MAX)}$$

Question 4   **(3 marks)**

The area enclosed between the curve $y = \frac{2}{3x-1}$, the lines $x = 1$ and $x = 3$ and the $x - axis$ can be written as $\log_e B$ square units. Calculate the value of B.

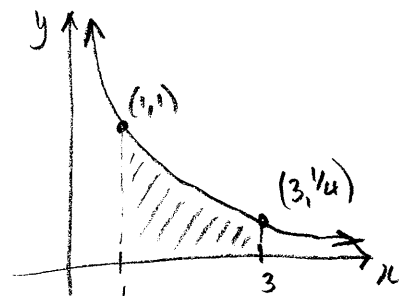$$\int_1^3 \frac{2}{3x-1}\, dx = \frac{2}{3}\int_1^3 \frac{3}{3x-1}\, dx$$

$$= \frac{2}{3}\left[\ln(3x-1)\right]_1^3 \quad \checkmark \text{ (Anti-diff)}$$

$$= \frac{2}{3}\left[\ln 8 - \ln 2\right]$$
$$\checkmark \text{ (sub in)}$$

$$= \frac{2}{3}\ln 4$$

$$= \ln 4^{2/3}$$

$$\therefore B = \boxed{4^{2/3}} \quad \checkmark \text{ (B-value)}$$

End of questions

6

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2020
TEST 4
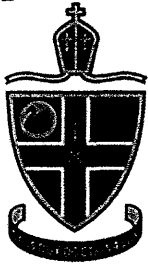
MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name　　　• SOLUTIONS •

Teacher's name

Time and marks available for this section
Reading time before commencing work:　　3 minutes
Working time for this section:　　30 minutes
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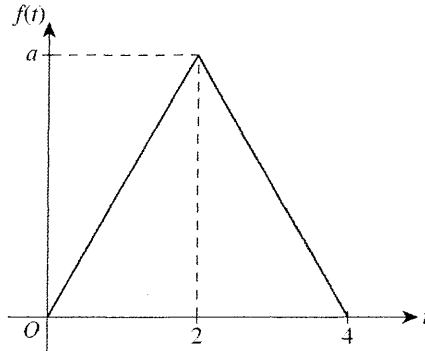
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Question 5 (5 marks)

The length of time, T minutes, that a Mathematics Methods student takes to answer an examination question is a random variable defined by the probability function whose graph is shown below.

(a) State the value of a so that f is a probability density function. (1 mark)

$$1 = \frac{1}{2}(4) \times a$$

$$\therefore \boxed{a = \frac{1}{2}} \checkmark$$

(b) Hence state the rule for the probability function f. (2 marks)

Line from $(0,0)$ to $(2, \frac{1}{2})$ $\Rightarrow f(t) = \frac{t}{4}$

" " $(2, \frac{1}{2})$ to $(4, 0)$ $\Rightarrow f(t) = 1 - \frac{t}{4}$

$$\therefore \quad f(t) = \begin{cases} \frac{t}{4} & \text{for} \quad 0 \le t \le 2 \\ 1 - \frac{t}{4} & \text{for} \quad 2 < t \le 4 \\ 0 & \text{elsewhere} \end{cases} \checkmark\checkmark$$

$\begin{bmatrix} -1 \text{ for each} \\ \text{missing or} \\ \text{incorrect} \\ \text{domain} \end{bmatrix}$

(c) Find $\Pr(T \le 1)$. (2 marks)

$$\Pr(T \le 1) = \int_0^1 \frac{t}{4}\, dt \quad \checkmark$$

$$= \boxed{\frac{1}{8}} \quad \checkmark$$

5

Question 6 **(2 marks)**

In a large population of fish, the proportion of angel fish is 0.25. Let $\hat{P}$ be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.
Find the smallest integer value of n such that the standard deviation of $\hat{P}$ is less than or equal to 0.01.

$$sd\ (\hat{P}) = \sqrt{\frac{P(1-P)}{n}}$$

$$0.01 = \sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{n}} \qquad \checkmark \quad \left(\text{Equates sd to } 0.01\right)$$

$$\therefore \quad n = 1875 \qquad \checkmark \quad \left(n \text{ value}\right)$$
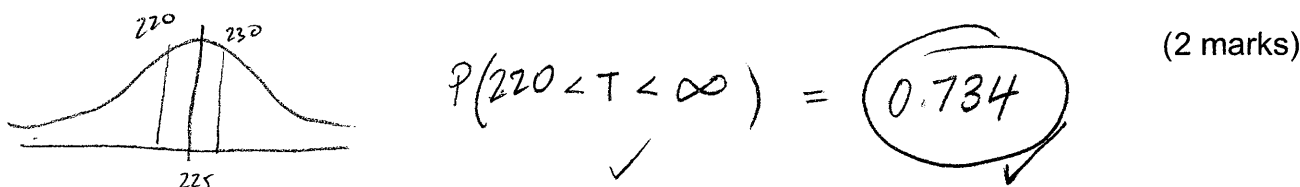
2

Question 7 (10 marks)

The travelling time for shuttle buses from train station A to train station B (Route AB) is found to be normally distributed with a mean arrival time of 3 hours and 45 minutes and standard deviation of 8 minutes.
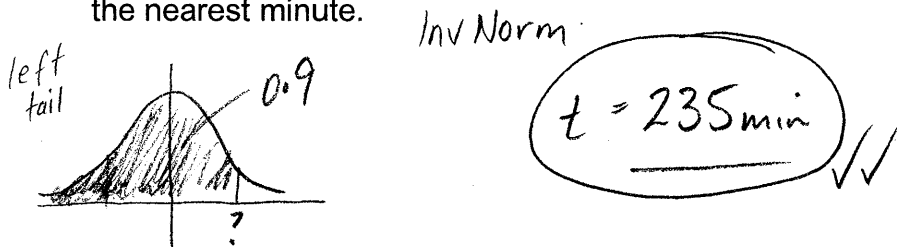
(a) Find the probability that a bus on Route AB will complete its journey in under 3 hours and 50 minutes. Give your answer correct to three decimal places.

(2 marks)

$\mu = 225 \, min$

$\sigma = 8 \, min$ ✓ $\quad T \sim N(225, 8^2), \quad P(T < 230) = \boxed{0.734} \; ✓ \, (3dp)$
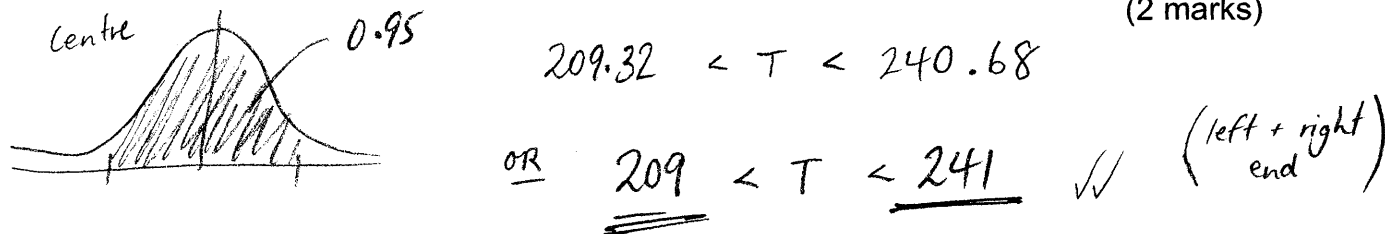
(parameters)

(b) Hence, find the probability that the same bus will be no earlier than 5 minutes from its mean arrival time. Give your answer correct to three decimal places.

(2 marks)

$P(220 < T < \infty) = \boxed{0.734}$ ✓

(c) 90% of the shuttles will complete the Route AB within t minutes. Find t correct to the nearest minute. (2 marks)

left tail

$Inv \, Norm$

$\boxed{t = 235 \, min}$ ✓✓

(d) Within what range of times could we be 95% confident of completing Route AB ?

(2 marks)

Centre   0.95

$209.32 < T < 240.68$

OR $\quad 209 < T < 241$ ✓✓ (left + right end)

A random sample of the travel time to complete Route AB is taken from the time sheets of 10 shuttles.

(e) What is the probability that of the 10 times selected, 6 of them will have completed the journey in under 3 hours and 50 minutes? Give your answer correct to four decimal places. (2 marks)

Binomial

$\binom{10}{6}(0.734)^6(0.266)^4 = \boxed{0.1644}$ ✓ ✓

10

Question 8 (3 marks)

The probability density function for a continuous random variable X has the rule

$$f(x) = \begin{cases} a\sin(\pi x) & for \quad 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

(a) Find the value of a. (1 mark)

Using ClassPad

$$\int_0^1 a\sin(\pi x)\, dx = 1.$$

$$\therefore \quad \boxed{a = \frac{\pi}{2}} \quad \checkmark \quad (1.57)$$

(b) Find $Pr(X \leq \frac{1}{4} | X \leq \frac{1}{2})$. (2 marks)

$$= \frac{P\left(X \leq \frac{1}{4} \cap X \leq \frac{1}{2}\right)}{P(X \leq \frac{1}{2})} \quad \checkmark \quad \Rightarrow \quad \frac{P\left(X \leq \frac{1}{4}\right)}{P\left(X \leq \frac{1}{2}\right)}$$

$$\frac{-\frac{\sqrt{2}}{4} + \frac{1}{2}}{\frac{1}{2}} = \frac{2 - \sqrt{2}}{4} \div 2 \quad \checkmark \quad \Rightarrow \quad \frac{2-\sqrt{2}}{8}$$

$$(0.2929)$$

Question 9 (3 marks)

Jo, a physiotherapist, is undertaking a study of her patients' exercising habits. She decides to ask every eighth patient about their exercise habits.

(a) What type of sample is this? (1 mark)

Systematic Sampling $\checkmark$

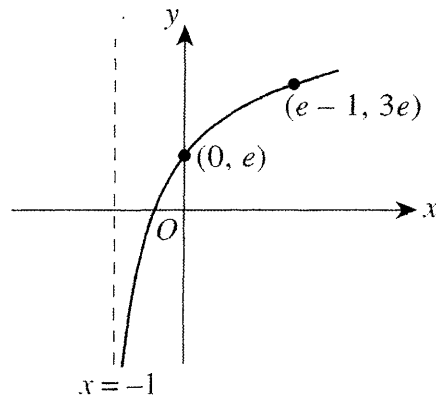(b) Is this method of sampling valid? Justify your answer. (2 marks)

YES, assuming that order of sampling is random.

(ANSW) (REASON)

See next page

Question 10   **(3 marks)**

The graph of $y = a \log_e(x - b) + c$ is shown below.

Calculate the exact values of b, c and a.

① Vertical asymptote $x - b = 0$

∴ $-1 - b = 0$

$\boxed{b = -1}$ ✓

② Sub in $(0, e)$ ⟹ $a \log_e (-b) + c$

As $b = -1$

$a \log_e 1 + c = e$

As $\log_e 1 = 0$

$\boxed{c = e}$ ✓

③ Sub in $(e-1, 3e)$

As $b = -1, c = e$ $3e = a \log_e (e - 1 + 1) + e$

$2e = a \log_e e$

5.436

$\boxed{a = 2e}$ ✓

3

Question 11 (4 marks)

During the COVID-19 pandemic Jeff's Gin Distillery that once produced ethanol for gin, now uses it to produce sanitizer. It fills bottles of sanitizer uniformly to between 245 ml and 253 ml.
The label on the bottle states that it holds 250ml.

(a) Determine the probability that a bottle selected randomly from the production line contains less than the labelled amount. (1 marks)

$$P(x < 250) = \frac{5}{8}$$
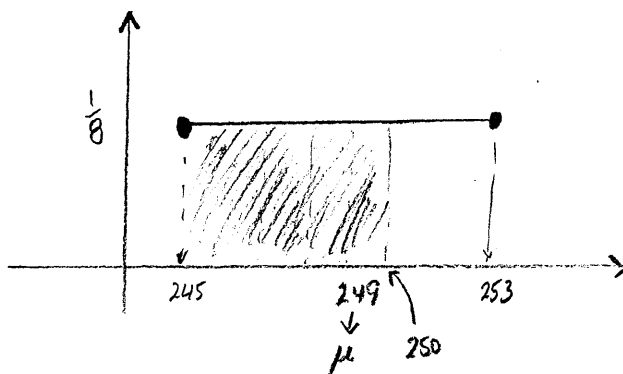
(b) Calculate the mean and standard deviation of the amount of sanitizer in the bottles. (3 marks)

$$\bar{x} = 249 \, ml \quad (\text{Uniform Dist}) \quad \checkmark \quad (\text{MEAN})$$

$$\sigma^2 = \int_{245}^{253} (x - 249)^2 \times \frac{1}{8} \, dx = 5\frac{1}{3} \quad \left(\frac{16}{3}\right)$$

$$\checkmark \quad (\text{VAR})$$

$$\therefore \sigma = \sqrt{var}$$

$$\sigma = \sqrt{5\frac{1}{3}}$$

$$\sigma = \frac{4\sqrt{3}}{3} \quad (2.31) \quad \checkmark \quad (\text{SD})$$

End of questions